Dynamic Polar Orientational Order in Active Media

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In conventional liquid crystals orientational order is described by the well-known quadrupolar order parameter $Q_{ij}= (S/2)(3n_i n_j - \delta_{ij})$ or the polar order parameter $P_i = P p_i$ depending on whether the system is macroscopically non-polar (nematic) or polar (polar nematic), respectively. These descriptions apply to equilibrium phases, where the non-zero order ($S \neq 0$ or $P \neq 0$) arises from a spontaneous symmetry breaking due to the (mean) preferred orientation of the molecules (static order). This orientation is a property of the thermodynamic ground state of those equilibrium phases. The dynamics of those systems then contain the rotation of the unit vector $\partial_t n_i$ or $\partial_t p_i$ (Goldstone mode) as well as the relaxation of order fluctuations $\partial_t \delta S$ or $\partial_t \delta P$ (the soft mode near a second order phase transition).

Over the last few years the collective dynamics of active systems has attracted increasing attention of the physics community. These are (predominantly) biological systems, where the entities can move by themselves. They are driven out of equilibrium, not by externally applied driving fields, but internally - typically by chemical reactions. As long as the internal driving forces are operating, the units of the active system move, and due to this motion an (usually orientationally) ordered spatial structure is created. Examples are schools of fish or flocks of birds, pattern forming growing bacteria (e.g. Proteus mirabilis), biological motors (myosin and actin), or generally suspensions of active particles. They are all internally driven by chemical reactions, e.g. the ATP to ADP consumption, the animals' metabolism, or any other feeding mechanism. If the internal drive stops, e.g. due to lack of nutrients, the system becomes passive, with no motion and no remaining localized collective spatial structures, anymore.
Often, such active systems have been described using the order parameters of equilibrium nematic or polar nematic phases. Here, we will take a different point of view [1] and describe the active structure as dynamic, created by the motion of active units. In the polar case, to which we will restrict ourselves here, the natural candidate for the variable to describe the dynamic structure is the velocity of the active entities, $F_i = F f_i$, where F is the absolute value of the mean velocity, and the unit vector f_i is the preferred direction. Since there is generally also a non-active background, we are left with two different velocities, the non-active one and the active one. The latter is finite even in a stationary state, $F = F_S \neq 0$ and there is no equilibrium active state ($F_S = 0$ means a non-active, passive state without order).

We derive the macroscopic equations for such a system and discuss novel static, reversible and irreversible cross-couplings connected to this second velocity. We find a normal mode structure quite different compared to the case of static order, as well as linear couplings between (active) flow and e.g. densities and concentrations due to the genuine two-fluid transport derivatives. On the other hand, we get, quite similar to the static case, a direct linear relation between the stress tensor and the structure tensor. This prominent "active" term is responsible for many active effects, meaning that our approach can describe those effects as well. In addition, we also deal with explicitly chiral systems, which are important for many biological systems. In particular, we find an active flow-induced heat current, which is specific for dynamic chiral polar order. A similar treatment of non-polar active systems has been given recently [2]. An application of the dynamic order concept to the front propagation of bacteria (Proteus mirabilis) [3] will be presented by D. Svensek.

References:
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